Math 4200 , Review 1

1. Show that $\sqrt{43}$ is an irrational number.

2. Write the following statements in the form $p \to q$. Find the contrapositive for each.

   a) When $f'(x) > 0$ for each x in an interval I , f is increasing on I .

   b) A necessary condition for a function to be differentiable at $x = a$ is for f to be continuous at $x = a$.

3. Construct a truth table for the statement $(\, p \text{ and } \sim r\,) \to (\, \sim q \text{ or } r\,)$.

4. A sequence $\{a_n\}$ is said to be a Cauchy sequence provided the following condition is true.

   For each $\varepsilon > 0$, there exists $N > 0$ such that for all positive integers m and n , if $m, n \geq N$ then $|\, a_n - a_m \,| < \varepsilon$.

   State the precise negation of this condition.

5. In a class with 30 students, how many ways can you select 5 students to serve on a committee given that three of the students refuse to serve together? (Each one of these three students is willing to serve but not if one of the others is selected.)

6. a) Suppose A, B, and C are sets. Show that $A \times (B - C) = (A \times B) - (A \times C)$.

b) Let R^+ denote the positive real numbers. Find $\bigcap_{\lambda \in R^+} (5, 13 + \lambda)$ and $\bigcup_{\lambda \in R^+} (-\infty, \lambda)$.

7. Show that the following sets are equivalent; that is, for each pair of sets, there exists a 1-1 correspondence between them.

a) $(0,1)$ *and* $(-4, 4)$

b) $(0,1)$ *and* $(-4, \infty)$

c) Q *and* J

8. Let S be the set of all points in the upper-half plane with integer coordinates. That is,

$S = \{ (x,y) :$ x is an integer and y is a positive integer$\}$. Show that S is equivalent to J.

9. Show that the set of all sequences of 3's and 7's is not a countably infinite set. Describe how this result will imply that the set of all real numbers is an uncountable set.

10. Prove: For each natural number n , $17^n - 12^n$ is divisible by 5 .